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Assignment II MTH 512 , Fall 2019

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QUESTION 1. Let $F : R^4 \to R^3$, be a linear transformation. $B = \{(1,0,2,0),(0,1,1,0),(0,0,1,1),(-1,0,0,1)\}$ and $B' = \{(1,1,0),(-1,1,0),(-1,-1,1)\}$ be basis for R^4 and R^3, respectively. Given $F(1,0,2,0) = (1,-1,-1)$, $F(0,1,1,0) = (-1,0,1)$, $F(0,0,1,1) = (-2,0,2)$ and $F(-1,0,0,1) = (0,-1,0)$.

(i) Find the matrix presentation of F with respect to B and B', $M_{B,B'}$. (i.e., $M_{B,B'}$ = "something", I want to see that "something", however to calculate that "something" use software calculator as on I-learn)

$$\underset{3\times 4}{M_{BB'}} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & -2 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -2 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & \frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & 1 & 2 & 0 \end{bmatrix}$$

(ii) USE (i) and find $[T(2,5,8,2)]_{B,B'}$

Note (again) write down clearly the steps, however use software calculator to do the actual calculation

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$$B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow B^{-1} = \begin{bmatrix} -1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 2 \\ -2 & -1 & 1 & -1 \end{bmatrix}$$

$$[2,5,8,2]_B = B^{-1}\begin{bmatrix} 2 \\ 5 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 5 \\ -3 \end{bmatrix}, \quad [T(-1,5,5,-3)]_{B'} = M_{B,B'}\begin{bmatrix} -1 \\ 5 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 16 \end{bmatrix}$$

(iii) Use (ii) and find $T(2,5,8,2)$.

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$$T(2,5,8,2) = B' \begin{bmatrix} 10 \\ 10 \\ 16 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 16 \end{bmatrix} = \begin{bmatrix} -16 \\ 4 \\ 16 \end{bmatrix}$$

(iv) Use (i) and find the standard matrix presentation. (I will not say it again, I want to see how you find M, actual calculations by software calculator)

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$$M = B' M_{B,B'} B^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & \frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & \frac{1}{2} & 1 & -\frac{1}{2} \\ -1 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 2 \\ -2 & -1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -4 & 3 & -5 \\ 3 & 2 & -2 & 2 \\ 5 & 4 & -3 & 5 \end{bmatrix}$$

QUESTION 2. Let $F : R^4 \to R^3$ such that $T(a_1, a_2, a_3, a_4) = (2a_1 + a_4, -a_3, 4a_1 + 2a_3 + a_4)$

(i) Write range(F) as span of some independent points.

$$T(a_1,a_2,a_3,a_4) = \{a_1(2,0,4) + a_2(0,0,0) + a_3(0,-1,2) + a_4(1,0,1) \mid a_1,a_2,a_3,a_4 \in \mathbb{R}\}.$$
$$= \text{span}\{(2,0,4),(0,0,0),(0,-1,2),(1,0,1)\}$$
$$= \text{span}\{\underbrace{(2,0,4)}_{Q_1},\underbrace{(0,-1,2)}_{Q_2},\underbrace{(1,0,1)}_{Q_3}\}.$$

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(ii) Write range(F) as span of orthogonal points

[For this particular question
$= \text{span}\{e_1, e_2, e_3\}$]

$$W_1 = Q_1 = (2,0,4)$$

$$W_2 = Q_2 - \frac{Q_2 \cdot W_1}{\|W_1\|^2} W_1 = (0,-1,2) - \frac{(0,-1,2)\cdot(2,0,4)}{\|(2,0,4)\|^2}(2,0,4) = \left(\frac{-4}{5}, -1, \frac{2}{5}\right).$$

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$$W_3 = Q_3 - \frac{Q_3 \cdot W_2}{\|W_2\|^2} W_2 - \frac{Q_3 \cdot W_1}{\|W_1\|} W_1$$

$$= (1,0,1) - \frac{(1,0,1)\cdot(-\frac{4}{5},-1,\frac{2}{5})}{\|(-\frac{4}{5},-1,\frac{2}{5})\|^2}\left(\frac{-4}{5},-1,\frac{2}{5}\right) - \frac{(1,0,1)\cdot(2,0,4)}{\|(2,0,4)\|^2}(2,0,4) = \left(\frac{2}{9}, \frac{-2}{9}, \frac{-1}{9}\right)$$

(iii) Does the point $(2,5,9)$ belong to Range(F)? Explain?

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$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 4 & 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix} \to \left[\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & 0 & 5 \\ 4 & 0 & 2 & 1 & 9 \end{array}\right] \to \begin{cases} a_1 = \frac{17}{2} \\ a_2 = \text{free variable} \\ a_3 = -5 \\ a_4 = -15 \end{cases}$$

$\therefore$ YES. because there is at least one point $\in \mathbb{R}^4$ such that $F(\text{at this point}) = (2,5,9)$.

$$\text{Sol. Set} = \left\{\left(\frac{17}{2}, a_2, -5, -15\right)\right\}$$

(iv) Write Z(F) as span of some independent points

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$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 4 & 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \left[\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 4 & 0 & 2 & 1 & 0 \end{array}\right] \to \begin{cases} a_1 = 0 \\ a_2 = \text{free variable} \\ a_3 = 0 \\ a_4 = 0 \end{cases}$$

$$\text{Sol. set} = \{(0, a_2, 0, 0) \mid a_2 \in \mathbb{R}\}$$
$$= \{a_2(0,1,0,0) \mid a_2 \in \mathbb{R}\}$$
$$= \text{span}\{(0,1,0,0)\}.$$

(v) Find the Standard matrix presentation of F.

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$$M = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 4 & 0 & 2 & 1 \end{bmatrix}$$

(vi) Use (V) and find $T(-2, 3, 6, 1)$

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$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 4 & 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ 6 \\ 1 \end{bmatrix} = -2\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 6\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}$$

QUESTION 3. Let $F : R^3 \to R^4$ such that $T(2, 0, 0) = (1, 1, 1, 1)$, $T(2, 2, 0) = (-2, -2, -2, -2)$, and $T(-1, -2, 1) \in Z(F)$.

- T is a Linear Transformation.

(i) Find the standard matrix presentation of F

$* \; T(e_1) = T(1,0,0) = T\left(\frac{1}{2}(2,0,0)\right) = \frac{1}{2}T(2,0,0) = \frac{1}{2}(1,1,1,1) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right).$

$* \; T(e_2) = T(0,1,0) = T\left(-\frac{1}{2}(2,0,0) + \frac{1}{2}(2,2,0)\right) = -\frac{1}{2}T(2,0,0) + \frac{1}{2}T(2,2,0)$

$= -\frac{1}{2}(1,1,1,1) + \frac{1}{2}(-2,-2,-2,-2) = \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) + (-1,-1,-1,-1) = \left(\frac{-3}{2}, \frac{-3}{2}, \frac{-3}{2}, \frac{-3}{2}\right)$

$* \; T(e_3) = T(0,0,1) = T\left(-\frac{1}{2}(2,0,0) + (2,2,0) + (-1,-2,1)\right) = -\frac{1}{2}T(2,0,0) + T(2,2,0) + T(-1,-2,1)$

$= -\frac{1}{2}(1,1,1,1) + (-2,-2,-2,-2) + (0,0,0) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) + (-2,-2,-2,-2) = \left(\frac{-5}{2}, \frac{-5}{2}, \frac{-5}{2}, \frac{-5}{2}\right)$

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$$\therefore M = \begin{array}{c} \begin{array}{ccc} T(e_1) & T(e_2) & T(e_3) \end{array} \\ \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \end{bmatrix} \end{array}$$

(ii) write range of F as span of some independent points.

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$$\text{Range}(F) = \text{span}\left\{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\right\}$$

(iii) Write Z(F) as span of some independent points.

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$$\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \to \left[\begin{array}{ccc|c} \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & 0 \end{array}\right] \to \begin{array}{l} a_1 = 3a_2 + 5a_3 \\ a_2 : \text{free variable} \\ a_3 : \text{free variable.} \end{array}$$

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$$\text{Sol. set} = \{(3a_2 + 5a_3, a_2, a_3) \mid a_2, a_3 \in \mathbb{R}\}$$
$$= \{a_2(3,1,0) + a_3(5,0,1) \mid a_2, a_3 \in \mathbb{R}\}$$
$$= \text{span}\{(3,1,0), (5,0,1)\}$$